1 The velocity of a model boat, $\mathbf{v}$ m s^{-1}, is given by

$$\mathbf{v} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} + t \begin{pmatrix} 6 \\ 8 \end{pmatrix},$$

where t is the time in seconds and the vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are east and north respectively.

(i) Show that when $t = 2.5$ the boat is travelling south-east (i.e. on a bearing of 135°). Calculate its speed at this time. [3]

The boat is at a point O when $t = 0$.

(ii) Calculate the bearing of the boat from O when $t = 2.5$. [4]

2 The acceleration of a particle of mass 4 kg is given by $\mathbf{a} = (9\mathbf{i} - 4t\mathbf{j})$ m s^{-2}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors and t is the time in seconds.

(i) Find the acceleration of the particle when $t = 0$ and also when $t = 3$. [1]

(ii) Calculate the force acting on the particle when $t = 3$. [1]

The particle has velocity $(4\mathbf{i} + 2\mathbf{j})$ m s^{-1} when $t = 1$.

(iii) Find an expression for the velocity of the particle at time t. [4]

3 The position vector, $\mathbf{r}$, of a particle of mass 4 kg at time t is given by

$$\mathbf{r} = t^2\mathbf{i} + (5t - 2t^2)\mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors, lengths are in metres and time is in seconds.

(i) Find an expression for the acceleration of the particle. [4]

The particle is subject to a force $\mathbf{F}$ and a force $12\mathbf{j}$ N.

(ii) Find $\mathbf{F}$. [3]

4 A ring is moving on a straight wire. Its velocity is v m s^{-1} at time t seconds after passing a point Q.

Model A for the motion of the ring gives the velocity-time graph for $0 \leqslant t \leqslant 6$ shown in Fig. 7.

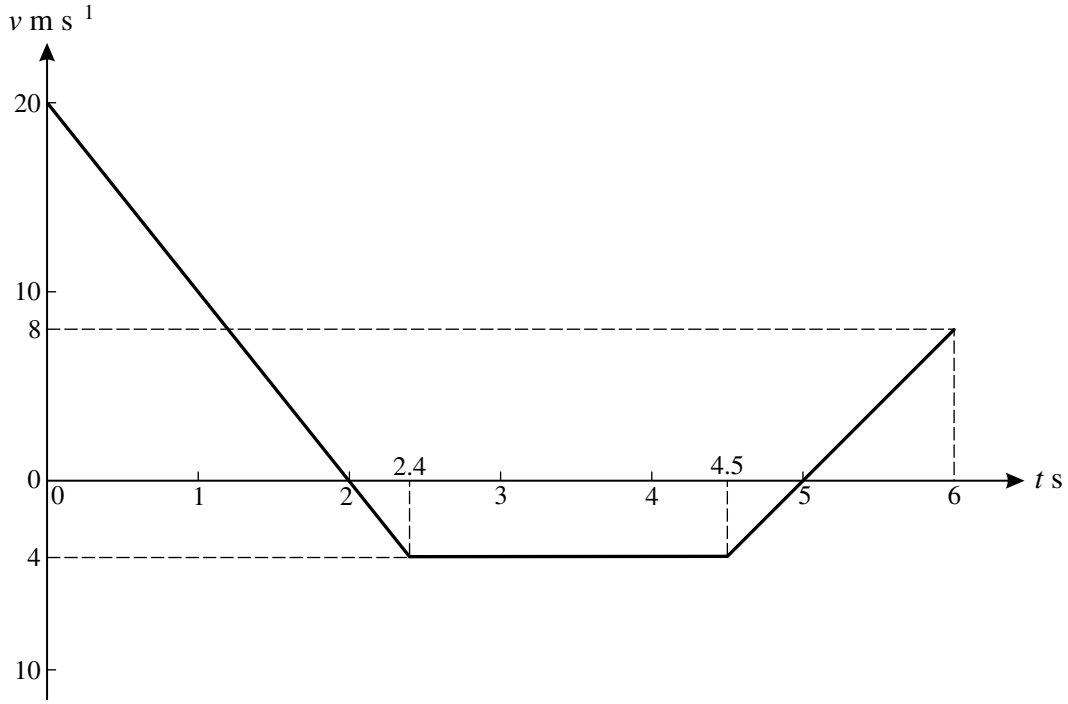

Fig. 7

Use model A to calculate the following.

(i) The acceleration of the ring when $t = 0.5$. **[2]**

(ii) The displacement of the ring from Q when

(*A*) $t = 2$,

(*B*) $t = 6$. **[5]**

In an alternative model B, the velocity of the ring is given by $v = 2t^2 - 14t + 20$ for $0 \leqslant t \leqslant 6$.

(iii) Calculate the acceleration of the ring at $t = 0.5$ as given by model B. **[3]**

(iv) Calculate by how much the models differ in their values for the least v in the time interval $0 \leqslant t \leqslant 6$. **[4]**

(v) Calculate the displacement of the ring from Q when $t = 6$ as given by model B. **[4]**